Magnetic collective mode in underdoped cuprates: a phenomenological analysis

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The dynamical spin susceptibility as relevant for underdoped cuprates is analysed within the memory-function (MeF) approach. A phenomenological damping function combined with a T-independent sum rule is used to describe the anomalous normal state and the resonant peak in the superconducting state, in particular its position and its relative intensity to the normal state. The relation with the random-phase approximation is discussed. The MeF method is generalized to the bilayer system in order to compare with inelastic neutron scattering experiments on $YBa_2Cu_3O_{6+x}$ which alows also for a quantitative comparison. In this context the problem of the missing integrated spectral intensity within the experimentally accessible energy window is also discussed.

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I. INTRODUCTION

Since the first observation of the resonant magnetic peak in superconducting (SC) optimally doped $YBa_2Cu_3O_{6+x}$ (YBCO),[1] the magnetic collective mode in cuprates and its role have been the subject of intensive experimental investigations, with the most direct information gained by the inelastic neutron scattering (INS). In YBCO it has been observed that with lower doping the resonant peak (RP) moves to lower energies, increasing at the same time in intensity.[2] In the last years, the intriguing feature of the RP hour-glass dispersion[3] has been in the focus. Recently, also the existence of an optical (even) resonant mode in the bilayer YBCO has been reconsidered.[4] It has been shown that the even mode can be well identified in underdoped (UD) regime, however with an essentially different intensity from the dominating odd mode. On the other hand, it seems clear that the RP has to be closely related to the magnetic response in the normal (N) state, which has been well resolved by INS in UD cuprates. The response is typically that of an overdamped collective mode, however with an anomalous ω/T scaling, being common both to UD YBCO[5] and single-layer $La_{2-x}Sr_xCuO_4$[6,7] cuprates.

Most theories, which address the RP and magnetic response in cuprates, are based on the treatment of the metallic system close to the antiferromagnetic (AFM) instability, describing the RP as a consequence of the AFM (over)damped soft mode in the N state and $d_{x^2-y^2}$ SC gap in the electron-hole excitation spectrum, leading to a sharp RP below T_c. Most frequently invoked is the RPA-like form for the dynamical susceptibility $\chi_{\mathbf{q}}(\omega)$, derived in various ways or argued via the Hubbard, t-J or analogous models.[8] The latter ascribe the RP to a weak excitonic mode below the electron-hole continuum and seem to account qualitatively for optimum doping and for the overdoped regime, in particular for the position and the peculiar downward dispersion of the RP.[9] The alternative approach, using the memory-function (MeF) description, recently introduced by the present authors,[10,11] gives analogous results in the latter cases, and in addition a more strongly pronounced upper dispersion.[12]

It is rather evident that the RPA-based theories for the magnetic response are less appropriate for the UD cuprates, even if taken in a broader sense as the phenomenological framework. In the first place, in the N state the usual RPA leads to T-independent and Fermi-liquid-like $\chi_{\mathbf{q}}(\omega)$, in contrast to the anomalous dynamics found by INS.[6,7] As discussed furtheron in more detail, the RPA-like description cannot account for a strong RP in UD cuprates and the spin-wave-like dispersion at higher energies, also observed in INS experiments.[13,14] On the other hand, the MeF approach to $\chi_{\mathbf{q}}(\omega)$ is able to give a unified description of the anomalous N-state as well as SC-state response, even more naturally in the UD regime. In this paper we will use the MeF formalism on the phenomenological level with some simplifying assumptions. It will be shown, that such an approach, generalized to a bilayer systems relevant for YBCO, can account for the T and doping evolution of $\chi_{\mathbf{q}}(\omega)$ in the UD regime, in particular for the intensity of the coherent RP, and its jump at the onset of SC in relation with its position. The framework is also used to make a quantitative comparison with INS results for UD YBCO.

The paper is organized as follows. In the following section we briefly sketch the MeF formalism (Sec. II.A) and then address the evolution of $\chi''_{\mathbf{q}}(\omega)$ with T, including the appearance of the RP and of its intensity relative to the sum rule $C_{\mathbf{q}}$ (Sec. II.B). Next we present a critical comparison of the MeF approach to the more commonly used RPA (Sec. II.C) and conclude the section with an extension of the MeF approach to a bilayer system, as appropriate to YBCO. In Section III we first use the available experimental data on $YBCO_{6.5}$ and $YBCO_{6.7}$ to extract the relevant parameters within MeF, and then present a quantitative comparison of the MeF approach to experiments. Conclusions are presented in Sec. IV.

II. MEMORY FUNCTION APPROACH

A. Formalism

Within the memory function formalism the dynamical spin susceptibility can be generally written as[10]

$$\chi_{\mathbf{q}}(\omega) = \frac{-\eta_{\mathbf{q}}}{\omega^2 + \omega M_{\mathbf{q}}(\omega) - \delta_{\mathbf{q}}}\,, \qquad \delta_{\mathbf{q}} = \omega_{\mathbf{q}}^2 = \frac{\eta_{\mathbf{q}}}{\chi^0_{\mathbf{q}}}, \quad (1)$$

where $\chi^0_{\mathbf{q}} = \chi_{\mathbf{q}}(\omega = 0)$ and the 'spin stiffness' $\eta_{\mathbf{q}} = -i\langle[S^z_{-\mathbf{q}} \dot{S}^z_{\mathbf{q}}]\rangle$ can be expressed with equal-time correlations for any microscopic model of interest and is in general a quantity only weakly dependent on $\mathbf{q}, T$ and even on hole doping c_h.[10] The latter is not the case for $\chi^0_{\mathbf{q}}$ or for the 'mode' frequency $\omega_{\mathbf{q}}$. Instead of searching for an explicit approximation for either of these quantities, we fix them via the fluctuation-dissipation sum rule,[10,11]

$$\frac{1}{\pi}\int_0^\infty d\omega \; \mathrm{cth}\frac{\omega}{2T}\chi''_{\mathbf{q}}(\omega) = \langle S^z_{-\mathbf{q}} S^z_{\mathbf{q}}\rangle = C_{\mathbf{q}}. \qquad (2)$$

The underlying idea is that the equal-time spin correlations $C_{\mathbf{q}}$ are much less sensitive on T. In particular, we conjecture that they do not change (significantly) at the N-SC transition, which we lateron verify for available YBCO data. Evidently, $C_{\mathbf{q}}$ is $\mathbf{q}$ and doping dependent, but note that in strongly correlated systems $C_{\mathbf{q}}$ is restricted by the total sum rule, $(1/N)\sum_{\mathbf{q}} C_{\mathbf{q}} = (1-c_h)/4$. In the following, we concentrate in the analysis to the commensurate $\mathbf{Q} = (\pi,\pi)$, so the relevant quantity is $C_{\mathbf{Q}}$.

Here, we do not intend to give a microscopic derivation for the damping function $\gamma_{\mathbf{q}}(\omega) = M''_{\mathbf{q}}(\omega)$, as performed within the t-J model.[10,11,12] Still, we use the observation that the low-ω damping within the N state of a doped AFM, being metallic and paramagnetic, is mainly due to the electron-hole excitations. If the Fermi surface of the doped system crosses the AFM zone boundary (as revealed by ARPES for most hole-doped cuprates), in the N state one gets $\gamma_{\mathbf{q}}(\omega \to 0) > 0$ for $\mathbf{q} \sim \mathbf{Q}$. Hence, we use as the phenomenological input in the N state the simplest approximation $\gamma_{\mathbf{q}}(\omega) = \gamma$, supported also by numerical calculations within the t-J model.[11]. The expression for $\chi_{\mathbf{q}}(\omega)$, Eq.(1), in the N state then reduces to a simple damped-oscillator form.

In the SC state the d-wave gap introduces a gap also into $\gamma_{\mathbf{q}}(\omega)$. Here, we are not interested in the dispersion of the RP,[9,12] but rather in particular $\mathbf{q} = \mathbf{Q}$. Hence, we assume at $T < T_c$

$$\gamma_{\mathbf{Q}}(\omega < \omega_c) = 0, \qquad \gamma_{\mathbf{Q}}(\omega > \omega_c) = \gamma, \qquad (3)$$

where the effective gap $\omega_c = 2\Delta(\mathbf{q}^*) < \Delta_0$ is given by the SC gap value at $\mathbf{q}^*$ where the FS crosses the AFM zone boundary. For the discussion of the RP the corresponding real part is essential,

$$M'_{\mathbf{Q}}(\omega) = \frac{\gamma}{\pi}\ln\left|\frac{\omega_c + \omega}{\omega_c - \omega}\right|, \qquad (4)$$

generating always an undamped excitonic-like RP at $\omega_r < \omega_c$.

B. Qualitative analysis within the MeF

The presented phenomenological theory has nontrivial consequences and predictions for the spin dynamics in cuprates, both for the N and thr SC state. It follows directly from Eq.(1) that the damped-oscillator form is well adapted to treat a collective mode in the UD regime close to the AFM instability. Here we focus on the $\mathbf{q} = \mathbf{Q}$ mode and take into account also the fact, that in the N state the latter is generally overdamped, requiring $\gamma > \omega_{\mathbf{Q}}$. Then, we get essentially two rather distinct regimes: a) if $\omega_{\mathbf{Q}} \gg \omega_c$, consistent with modest $C_{\mathbf{Q}} < 1$, the N-state $\chi''_{\mathbf{Q}}(\omega)$ is very broad leading to a weak excitonic-like RP at $\omega_r \lesssim \omega_c$. This is the situation corresponding to optimum-doped or overdoped cuprates. b) When $\omega_{\mathbf{Q}}$ is close or even below ω_c, requiring $C_{\mathbf{Q}} > 1$, there is a pronounced low-ω response already in the N state, transforming into a strong RP in the SC phase, exhausting a substantial part of the sum rule $C_{\mathbf{Q}}$.

In Fig. 1 we present typical $\chi''_{\mathbf{Q}}(\omega)$, corresponding to the UD regime. We fix the parameters from our knowledge of the results within the t-J model,[10] as relevant for cuprates. For a weakly doped AFM we adopt $\eta \sim 2J \sim 0.6t$ (note that $t \sim 400$ meV), the value for an undoped AFM. In the UD regime it is essential that the damping is small, $\gamma < t$, both to get reasonable low-ω response as well as underdamped spin waves at higher ω. Here, we assume $\gamma = 0.2t$, while the SC parameters are chosen in accordance with data for UD cuprates, e.g., $\omega_c = 0.15\ t$, $T_c = 0.05\ t$. Then $C_{\mathbf{Q}}$ enters into $\chi''_{\mathbf{Q}}(\omega)$ as the only "free" parameter. In spite of simplifications, we observe in Fig. 1(a) several features consistent with experiments: a) the mode is overdamped in the N state with a Lorentzian form $\chi''_{\mathbf{Q}}(\omega) \propto \omega/(\omega^2 + \Gamma^2)$ and a peak $\Gamma \propto T$ shifting with T, being the signature of the anomalous ω/T scaling, observed in UD cuprates.[6,7] b) Already at $T > T_c$ the peak Γ is below ω_c, and in fact close to the position of the RP, which is not just a coincidence. c) At $T < T_c$ there appears a strong RP, exhausting a large part of the sum rule $C_{\mathbf{Q}}$. Also it is well shifted from the effective gap, i.e., $\Delta\omega_r = \omega_c - \omega_r$ is not small. The presented case is very close to the actual INS results for YBCO at $x = 0.5 - 0.7$,[2,5,13] as discussed lateron.

We define the intensities as

$$I_{\mathbf{Q}} = \int d\omega \chi''_{\mathbf{Q}}(\omega). \qquad (5)$$

In Fig. 2 we follow the RP intensity I_r, as a function of the relative RP position $\Delta\omega_r/\omega_c$, where we vary $C_{\mathbf{Q}}$ and fix the other parameters. In this way, we cover the span between the weak excitonic RP (small $C_{\mathbf{Q}}$) up to a pronounced AFM soft mode (large $C_{\mathbf{Q}}$). In INS experiments the strength of the RP is usually presented as the difference between the SC- and N-state response,[5] i.e., as

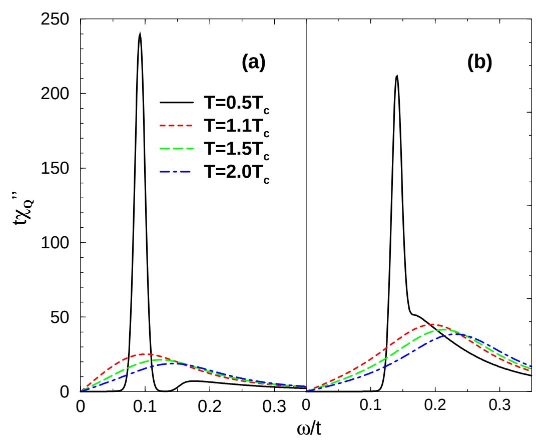

Figure 1: (Color online) Dynamical spin susceptibilities $\chi''_{\mathbf{Q}}(\omega)$ for different T above and below T_c, with other parameters fixed as given in the text. (a) $C_{\mathbf{Q}} = 2$, and (b) $C_{\mathbf{Q}} = 1$. Spectra are additionally broadened with $\delta = 0.01\ t$.

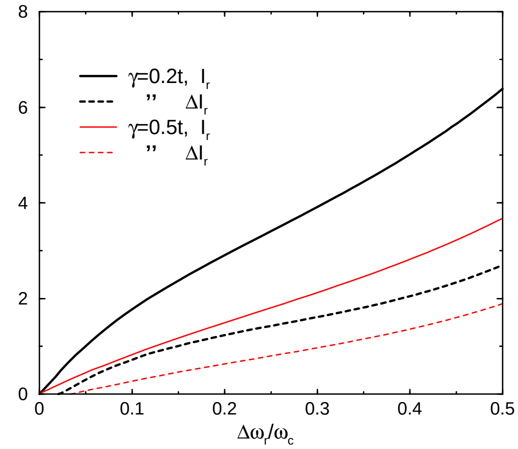

Figure 2: (Color online) Resonant peak intensities I_r (full lines) and ΔI_r (dashed lines), without and with N-state intensity subtracted, vs. relative position $\Delta\omega_r/\omega_c$, for two values of γ and varying $C_{\mathbf{Q}}$.

$$\Delta I_r = I_r(T \sim 0) - I_r(T \gtrsim T_c), \tag{6}$$

hence we plot in Fig. 2 also ΔI_r, with subtracted N-state intensity at $T > T_c$ and $\chi''_{\mathbf{Q}}(\omega)$ integrated in the range $0 < \omega < \omega_c$. As discussed in Sec. IID, such a measure can be directly compared with the recent INS experiments.[4] As noticed elsewhere,[4,16] it follows from the singular behavior of $M'(\mathbf{Q},\omega)$, Eq. (4), that $I_r \propto \Delta\omega_r$, valid in the regime of weak RP, $\Delta\omega_r \ll \omega_c$. From Fig. 2 one infers that a nearly linear dependence remains valid well beyond this limit. Also the RP enhancement ΔI_r shows similar dependence although overall reduced. Note that in the extreme case ΔI_r can become even slightly negative. It is also worth observing that ΔI_r vs. $\Delta\omega_r$ is only weakly dependent on γ, which might explain why YBCO with different doping x can be plotted roughly on a unique curve.[4]

C. Comparison with RPA

Let us also comment on the similarities and differences with the more usual RPA-like representation

$$\chi_{\mathbf{q}}(\omega) = [\tilde{\chi}_{\mathbf{q}}(\omega)^{-1} - \tilde{J}_{\mathbf{q}}]^{-1}, \tag{7}$$

where in the SC state at $T = 0$

$$\tilde{\chi}_{\mathbf{q}}(\omega) = -\frac{1}{N}\sum_{\mathbf{k}}\Big(1 - \frac{\tilde{\epsilon}_{\mathbf{k+q}}\tilde{\epsilon}_{\mathbf{k}} + \Delta_{\mathbf{k+q}}\Delta_{\mathbf{k}}}{E_{\mathbf{k+q}}E_{\mathbf{k}}}\Big) \times \frac{E_{\mathbf{k+q}} + E_{\mathbf{k}}}{\omega^2 - (E_{\mathbf{k+q}} + E_{\mathbf{k}})^2}, \tag{8}$$

with $E_{\mathbf{k}} = \sqrt{\tilde{\epsilon}_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2}$, whereas in the N state $\tilde{\chi}_{\mathbf{q}}$ corresponds to the usual Lindhard function, and $\tilde{\epsilon}_{\mathbf{k}} = -2\tilde{t}[\cos(k_x a) + \cos(k_y a)] - 4\tilde{t}'\cos(k_x a)\cos(k_y a) - \mu$ is the quasiparticle (QP) dispersion. The form, Eq.(7), has been derived in several ways or postulated based on microscopic models such as the Hubbard model and the t-J model.[8,9] We treat it as a phenomenological expression where parameters $\tilde{J}_{\mathbf{Q}}, \tilde{t}, \tilde{t}'$ are T independent, while still possibly doping dependent.

In spite of the apparent different forms, RPA and MeF, Eq. (1), can be related at low ω. If the QP band crosses the AFM zone boundary, we get in the N state $\tilde{\chi}''_{\mathbf{Q}}(\omega \to 0) = \tilde{\Gamma}_{\mathbf{Q}}\omega$. Using the relation

$$(\tilde{\chi}_{\mathbf{Q}}(\omega)^{-1} - \tilde{J}_{\mathbf{Q}})\eta_{\mathbf{Q}} = \delta_{\mathbf{Q}} - \omega^2 - \omega M_{\mathbf{Q}}(\omega), \tag{9}$$

it follows

$$\gamma = M''_{\mathbf{Q}}(\omega \to 0) = \eta\tilde{\Gamma}_{\mathbf{Q}}/(\tilde{\chi}^0_{\mathbf{Q}})^2, \tag{10}$$

while $\eta_{\mathbf{Q}} = -[\omega^2\tilde{\chi}_{\mathbf{Q}}]_{\omega\to\infty} \propto \tilde{t}$. Since in the SC state $\tilde{\chi}''_{\mathbf{Q}}(\omega < \omega_c) = 0$, the RPA result is also at $T < T_c$ formally close to our phenomenological MeF approach.

Nevertheless, the qualitative and even more quantitative differences become very pronounced in the UD regime approaching the undoped AFM: a) in the N state $\chi''_{\mathbf{q}}(\omega)$, Eq. (7), is essentially T independent and cannot account for the N-state anomalous scaling. b) For all reasonable QP bands, the effective γ within the RPA obtained from Eq. (10) is very large, i.e., typically $\gamma > t$. This prevents underdamped spin waves even at large ω. c) The intensities I_r of the RP are generally small, and in particular the N-SC difference $\Delta I_r \sim 0$.

For illustration, we present in Fig. 3 characteristic results within the RPA approach in the N and SC state $T \sim 0$, which correspond as close as possible to the regime of MeF results in Fig. 1. We choose the same ω_c, while $\tilde{t} = 3\tilde{t}' = 0.3t$. Data in Fig. 3(a) correspond

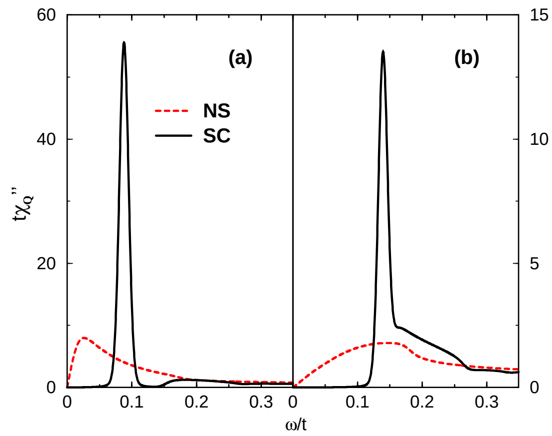

Figure 3: (Color online)$\chi''_{\mathbf{Q}}(\omega)$ as evaluated within the RPA in the N state and SC state, respectively. (a) $\tilde{J}_{\mathbf{Q}} = 0.95\tilde{J}^c_{\mathbf{Q}}$, (b) $\tilde{J}_{\mathbf{Q}} = 0.75\tilde{J}^c_{\mathbf{Q}}$.

to $\tilde{J}_{\mathbf{Q}} = 0.95\tilde{J}^c_{\mathbf{Q}}$, i.e., close to the AFM instability. Although the RP is quite intense in this case, exhausting 60% of the sum rule, the respective $I_r \sim 1.0$ is considerably smaller in comparison to both the MeF case (Figs. 1(a),2) and the INS data given later (see Table I). On entering the SC state $C_{\mathbf{Q}}$ exhibits a drop in magnitude, which may be quite substantial for $\tilde{J}_{\mathbf{Q}}$ close to $\tilde{J}^c_{\mathbf{Q}}$, again in contrast with experiments. Similarly, for $\tilde{J}_{\mathbf{Q}} = 0.75\tilde{J}^c_{\mathbf{Q}}$ (Fig. 3(b)) where the RP intensity still accounts for $\sim 20\%$ of the sum rule, we get $I_r \sim 0.25$, one order of magnitude smaller than the experimental data. Also note that ΔI_r remains negligeable ($\lesssim 0.3$) for all $\tilde{J}_{\mathbf{Q}} < \tilde{J}^c_{\mathbf{Q}}$.

D. Coupled-layers analysis

In order to be able to describe more quantitatively the INS results for YBCO, which is a bilayer system, we generalize the MeF approach to two coupled layers. We assume that the interaction between layers is only via an isotropic exchange[16]

$$H_{12} = J_\perp \sum_i \mathbf{S}_{i1} \cdot \mathbf{S}_{i2}. \tag{11}$$

Susceptibility can be defined as a matrix $\chi^{ll'}_{\mathbf{q}}(\omega)$, as well as other quantities, $\delta^{ll'}_{\mathbf{q}}, \eta^{ll'}_{\mathbf{q}}$ etc. The plausible assumption we use here is that the dominant damping due to the electron-hole excitations can originate only from the intraplanar hopping, therefore $M^{12} = 0$. Taking into account the symmetry $\chi^{11} = \chi^{22}$ etc. and defining even and odd functions, respectively, $\chi^{e,o}_{\mathbf{q}} = \chi^{11}_{\mathbf{q}} \pm \chi^{12}_{\mathbf{q}}$ etc., we can write in analogy with Eq. (1)

$$\chi^{e,o}_{\mathbf{q}} = \frac{-\eta^{e,o}_{\mathbf{q}}}{\omega^2 + \omega M_{\mathbf{q}}(\omega) - \delta^{e,o}_{\mathbf{q}}}, \tag{12}$$

Table I: Values extracted from INS results for UD YBCO (odd mode). $C_{\mathbf{Q}}$ and $I^{exp}(= 4I\mu_B^2)$, are estimated from the data published in [a] Ref. 2, [b] Ref. 13, and [c] Ref. 5.

	$YBCO_{6.5}$			$YBCO_{6.7}$ [c]		
T [K]	5[a]	85[b]	100[a]	12	70	200
$C_{\mathbf{Q}}$[1/f.u.]	2.1	1.6	1.9	1.4	1.4	1.5
$I^{exp}_{\mathbf{Q}}[\mu_B^2/\mathrm{f.u.}]$	26.4	18.0	11.3	16.9	14.9	8.8
$I^{exp}_r[\mu_B^2/\mathrm{f.u.}]$	19[b]			12.6		

where $\delta^{e,o}_{\mathbf{q}}$ are related $C^{e,o}_{\mathbf{q}}$ via the sum rules as before (Eq.(2)). Also, $\eta^{e,o}_{\mathbf{q}}$ can be explicitly expressed for the above interplane coupling H_{12}, and we get

$$\Delta\eta = \eta^o_{\mathbf{Q}} - \eta^e_{\mathbf{Q}} \propto J_\perp \langle \mathbf{S}_{1i} \cdot \mathbf{S}_{2i} \rangle < (J_\perp/J)\eta. \tag{13}$$

The estimates in the literature are $J_\perp/J \sim 0.1$,[15] therefore to the lowest approximation we can neglect the difference, and $\eta^{e,o}_{\mathbf{Q}} \sim \eta$. The only appreciable source of distinction between the even and odd response is therefore $C^o_{\mathbf{Q}} > C^e_{\mathbf{Q}}$. The latter difference can become substantial in UD cuprates which are close to the AFM instability, where the soft mode is $\omega^o_{\mathbf{Q}}$ while $\omega^e_{\mathbf{Q}} \propto \sqrt{J_\perp J}$ remains gapped.[15]

With the above simplifications we can apply previous single-plane results directly to bilayer YBCO. In particular, in Fig. 2 results for $I_r, \Delta I_r$ vs. $\Delta\omega_r$ for even and odd mode should fall on the same quasi-linear curve (for same γ, η), corresponding to different $C_{\mathbf{Q}}$. E.g., we note that $\Delta\omega_r/\omega_c = 0.05, 0.3$ correspond (for $\gamma = 0.2\ t$) to $C_{\mathbf{Q}} = 1.2, 1.75$, respectively, being qualitatively consistent with INS experiments.[5]

III. MeF APPROACH vs EXPERIMENTS

Let us use the existing quantitative INS data for YBCO to check the relevance of the presented MeF analysis. First, the spectra for YBCO with $x = 0.5$[13] can be used to extract directly γ. Namely, at higher $\omega \sim 80$ meV the spin-wave branches become underdamped and well resolved. From the width we can then estimate $\gamma \sim 70$ meV $\sim 0.2t$, the value assumed in our presentation in Figs. 1(a),2(a). We notice that such low γ (as well as spin waves) is essentially impossible within the RPA approach.

Next, we extract data for the intensities $I_{\mathbf{Q}}$ and the sum rule $C_{\mathbf{Q}}$, Eq. (2), for UD YBCO, both in the N and the SC state. To our knowledge such results are only available for $x = 0.7$[5] and $x = 0.5$.[2] Clearly, the spectra we can only integrate in the measured window $\omega < 70$ meV, so results the represent a lower estimate.

In Table I we present data for the odd mode, where the low-ω contribution is dominant. The important message is that $C_{\mathbf{Q}}$ is indeed quite T-independent and in particular nearly conserved at the N-SC transition, being the essential assumption within our MeF approach.

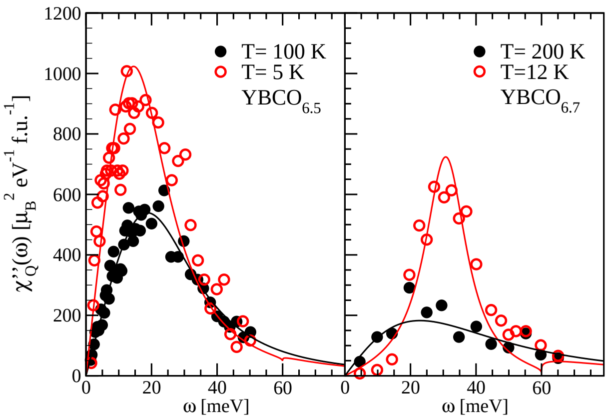

Figure 4: (Color online) MeF fit to INS data for $\chi''_{\mathbf{Q}}(\omega)$ (in absolute units) as given in Refs.2,5 Parameters are given in Table II.

Table II: Values adopted in fitting INS data for UD YBCO (odd mode) as presented in Fig. 4.

	$YBCO_{6.5}$		$YBCO_{6.7}$	
T [K]	5	100	12	200
$\eta_{\mathbf{Q}}$ [meV]	130	120	130	120
γ [meV]	35	40	60	85
$\omega_{\mathbf{Q}}$[meV]	20	26	38	40
γ_c [meV]	30		23	
$C_{\mathbf{Q}}$[1/f.u.]	2.1	2.1	1.2	1.4

Also, (theoretical) intensities $I^o_{\mathbf{Q}}, I^o_r$ are both large and comparable with MeF results in Fig. 2.

Finally, we present an attempt of a quantitative fit of the same INS data[2,5] using our phenomenological description as presented in Sec. IIA. We omit the $YBCO_{6.7}$ data at $T = 85$ K, which is just above T_c as there seems to be already an indication for a reduced N-state damping, since the response appears already underdamped. In adjusting the MeF form to the INS data we adopt the same form for $M_{\mathbf{Q}}(\omega)$ as in Sec. IIA, but for $T < T_c$ we keep $\gamma_{\mathbf{Q}}(\omega < \omega_c) = \gamma_0$ finite since experimentally the RP for T well below T_c seems not to be resolution limited,i.e., the RP has a finite width. Thus:

$$M''_{\mathbf{Q}}(\omega) = \gamma_0 \theta(\omega_c - \omega) + \gamma \theta(\omega - \omega_c)\,, \qquad (14)$$

$$M'_{\mathbf{Q}}(\omega) = \frac{\gamma - \gamma_0}{\pi} \ln\left|\frac{\omega_c + \omega}{\omega_c - \omega}\right|\,, \qquad (15)$$

where $\theta(x)$ is the step function. In adjusting the MeF form we try to keep $C_{\mathbf{Q}}, \eta = \eta_{\mathbf{Q}}, \gamma$ as T-independent as possible. As before, we fix the effective SC gap at $\omega_c \sim 60$ meV, consistent with experimental analysis[4]. The resulting parameters are listed in Table II, while the fits are presented in Fig. 4.

Overall, the fits appear quite satisfactory provided, however, that one chooses $\eta \sim 120 - 130$ meV $\sim J$. Essentially, η normalizes the intensities. While theoretically (from model calculations) $\eta_{th} \sim 2J$ is a very robust quantity, there are several reasons why η_{th} becomes renormalized. We are not dealing with the whole energy spectrum of spin fluctuations, but only with the low-frequency $\omega < J$ part. It is plausible, although theoretically not well explored, that there is a substantial high-frequency $\omega > J$ dynamics, in particular increasing with doping. This is clearly inferred from spectra obtained in numerical simulations on finite clusters for the t-J model at finite temperature,[18] where at low T and low to moderate doping a rather distinctive separation of energy scales for $\omega \lesssim J$ and $\omega \gtrsim 2\tilde{t}$ occurs. An enhancement of the sum rule $C_{\mathbf{Q}}$ with respect to the low-ω region should then occur. But η should become enhanced even more since, being the first frequency moment of $\chi''_{\mathbf{Q}}(\omega)$, it is more susceptible to high-ω tails in $\chi''_{\mathbf{Q}}(\omega)$. Likewise, a partial reduction of the effective fluctuating spin would also result in a smaller η. In any case, the same question already seems to be present when interpreting the INS results for undoped AFM where a reduced magnon intensity is observed through a renormalization factor $Z_\chi \sim 0.5$.[19]

Formally, a renormalization effect can be easily incorporated into the MeF analysis by adding an effective damping above some threshold $\omega^* > J$, assuming $M''_{\mathbf{Q}}(\omega > \omega^*) \sim \gamma^* \gg \gamma$ and

$$\Delta M'_{\mathbf{Q}}(\omega) = \frac{\Delta\gamma}{\pi} \ln\left|\frac{\omega^* + \omega}{\omega^* - \omega}\right| \sim \frac{2\Delta\gamma}{\pi\omega^*}\omega\,. \qquad (16)$$

where $\Delta\gamma = \gamma^* - \gamma$. Such a term would renormalize η for $\omega \ll \omega^*$

$$\eta \to \tilde{\eta} = \eta[1 + \frac{2\Delta\gamma}{\pi\omega^*}]^{-1}\,, \qquad (17)$$

which could explain the experimentally determined Z_χ. Note, however, that a separation of energy scales as mentioned above would also lead to a similar result, since a suppression of $\chi''_{\mathbf{q}}(\omega)$ in the energy interval $\omega^* < \omega < 2\tilde{t}$ is directly related to a local stepwise enhancement of damping within the same energy interval, i.e., $M''_{\mathbf{q}}(\omega^* < \omega < 2\tilde{t}) \sim \Delta\gamma$.

IV. CONCLUSIONS

To summarize, we have presented a phenomenological analysis of the magnetic collective mode in UD cuprates within the MeF approach and have compared it with frequently used RPA in this context. Both approaches give some qualitatitvely similar results for the RP, in particular $I_r \propto \Delta\omega_r$.[4,16] However, there are also clear differences. Due to large damping $\gamma \sim t$, the RPA cannot capture the high-energy magnons,[13,14] while these are easily reproduced within the MeF approach. Within RPA the RP intensities are too small in comparison to experiments, while $C_{\mathbf{Q}}$ drops at the N-SC transition, in contrast with experiments as analysed in Table I.

On the other hand, MeF shows in the N state an anomalous ω/T scaling of $\chi''_{\mathbf{Q}}(\omega)$, only being interrupted by the onset of SC, whereby the RP intensities I_r close to experimental (see Table I) can be easily reproduced. However, there is significant discrepancy between ΔI_r and $\Delta I_r^{exp}/4\mu_B^2$ as the latter is quite small. In fact, in underdoped cuprates ΔI_r^{exp} might not be as relevant a quantity for it compares the RP (integrated) intensity below T_c with the N-state intensity just above T_c, where the appearance of a pseudogap can already reduce damping γ and induce a RP-like response.[5,13,17]

Generalizing the MeF approach to a bilayer system as appropriate for YBCO we have shown that the RP intensities in both channels, i.e., $I_r^{e,o}$ should fall on the same quasi-linear curve vs. $\Delta\omega_r$, in agreement with experiments.[4]

Finally, we have performed a quantitative fit within the MeF approach, which agrees well with the INS data, provided that η at low ω is considerably renormalized.

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